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An analytical study of ion-acoustic solitary waves in a plasma consisting of two-temperature electrons and warm drift ions

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Abstract
Propagation of ion-acoustic waves in a multicomponent plasma consisting of warm positive ions with stream velocity and two-temperature electrons has been theoretically investigated. New analytical solutions with sufficient and necessary conditions are obtained for the existence of ion-acoustic solitary waves in the plasma. It is seen that the drifting ions and two temperature electrons have key roles on the formation of ion-acoustic solitary waves in a multicomponent plasma. The critical values of the ion temperature and phase velocity of the solitary waves have been numerically estimated, which would be applicable to various physical situations.

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1. Introduction

Studies of nonlinear characteristics of ion-acoustic waves in plasma are the subject of intense research during the past few years. Many interesting results are reported by various authors working in the field of solitary waves, shocks, breaking of waves, self-focusing, modulational instabilities etc which have wide applications to various physical situations in laboratory and space plasma. In recent years, much attention has been paid to the study of ion-acoustic solitary waves and double layers in plasma consisting of electrons at two different temperatures. These are observed in space [1] and also found in double-plasma (DP) machines [2], hot cathode discharges [3] and thermonuclear plasma [4]. Jones *et al* [5] were the first to investigate the ion-acoustic waves in a plasma consisting of ions and two types of electrons with different thermal effects. They considered a simple hot cathode discharge plasma where hot electrons were produced due to filament heating. Assuming the electrons of two Maxwellian distributions, Jones *et al* [5] derived the linear dispersion relation for the ion-acoustic waves in the presence of two electron temperatures. They observed that even a small percentage of cold electron concentration may predominate the ion acoustic speed, especially if the difference between the two electron temperatures increases. The theoretically predicted phase velocity of the wave was found to be in good agreement with the experimentally measured value. Goswami and Buti [6] following the works of Jones *et al* [5] derived the Korteweg–deVries (K–dV) equation for a cold ion plasma containing double Maxwellian electrons using the reductive perturbation method and obtained the corresponding ion-acoustic solitary wave solution. It was observed that the existence of the domain in an ion-acoustic solitary wave is rather limited by the respective concentrations and temperatures of the two electron components according to the theory of weakly nonlinear solutions. They concluded that the effect of cold electrons increases with the temperature of hot electrons which produce less dispersion to inhibit solitary wave formation. Following Goswami and Buti [6], several authors [7–16] investigated the solitons and other aspects of ion-acoustic waves in two-electron temperature plasmas and obtained important results which are applicable to various physical situations. The effects of two-temperature electrons on ion-acoustic waves described by Barthomier *et al* [17] are very interesting. They showed that the main characteristics of ion-acoustic solitary waves and weak double layers observed by the Swedish satellite Viking can be reproduced assuming the presence of two electron components in the auroral plasma. With the help of Sagdeev potential they have shown the characteristics of the ion-acoustic solitary waves excited in such a plasma. From their study, it was observed that the interactions between the hot and the cold electron component in the presence of a finite ion temperature produce rarefaction of the localized density. Such nonlinear structures exist in a more extended range of plasma parameters than that previously studied in the small amplitude limiting case using the K–dV equation. They have also found that the density of the cold population must always be smaller than the hot one, where the hot-to-cold temperature ratio must be greater than $\sim$10. The characteristics of these structures are quite different from those obtained in the small amplitude limiting case and can better reproduce the Viking observations in terms of their velocity, width and amplitude scales. Subsequently, Kourakis and Shukla [18] carried out the theoretical and numerical studies for the nonlinear amplitude modulation of ion-acoustic waves propagating in an unmagnetized, collisionless, three-component plasma composed of inertial positive ions moving in a background of two thermalized electron populations. They considered the perturbations oblique to the carrier wave propagation direction. It was shown from the stability analysis based on a nonlinear Schrödinger-type equation that the wave may become unstable and the stability criteria depend on the angle between the modulation and propagation directions. Moreover, different types of localized excitations (envelope solitary waves) were shown to exist. The results are in qualitative agreement with satellite observations in the magnetosphere. Another interesting result is found in the case of a plasma consisting of nonisothermal electrons. In the presence of resonant electrons, the plasma behaves nonisothermally. The resonant electrons strongly interact with the wave during its evolution and therefore cannot be treated assuming the Boltzmann distribution for the electron density as considered in an isothermal plasma. Schamel [19, 20] first considered the nonisothermality of electrons in a plasma and showed that an ion-acoustic wave in the lowest order has a new profile instead of the usual profile. Later, Das *et al* [21] and the other authors

[22, 23] assumed Schamel's plasma model and investigated the effects of nonisothermality of two-temperature electrons on the formation of ion-acoustic solitary waves.

But, the exact solution of the nonlinear equation for the ion-acoustic solitary waves in multicomponent plasma consisting of positive ions and electrons (isothermal or nonisothermal) at two different temperatures has not yet been obtained in the previous works. Recently, Ghosh *et al* [24] have obtained the exact solution including some necessary and sufficient conditions for the existence of solitary waves in a magnetized collisionless plasma consisting of positive ions and single temperature electrons. As the presence of two-temperature electrons in plasma gives some interesting characteristics on the solitary waves, the present paper deals with an analytical study to obtain some necessary and sufficient conditions for the existence of ion-acoustic solitary wave in a plasma consisting of electrons at two different temperatures and warm drift ions. The paper is organized as follows: in section 2, the basic equations and the formulation of the problem are described. In section 3, the necessary and sufficient conditions for the existence of ion-acoustic solitary waves in a plasma consisting of two-temperature electrons are obtained. Numerical estimations are made for the critical values of the phase velocity of the ion-acoustic solitary waves in a model plasma and the results are graphically discussed in section 4.

2. Formulation

We consider a collisionless, unmagnetized and fully ionized plasma consisting of warm drift positive ions and two populations of Boltzmann electrons which are separately in thermal equilibrium. The normalized basic equations in one dimension governing such a system are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\sigma}{n}\frac{\partial p}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} = 0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n. \tag{4}$$

The equations are dimensionless, where

$$n_e = n_{ec} + n_{eh}, \qquad n_{ec} = \mu\, \mathrm{e}^{(\phi/\mu+\nu\beta)}, \qquad n_{eh} = \nu\, \mathrm{e}^{(\beta\phi/\mu+\nu\beta)}, \qquad \mu + \nu = 1,$$

$$\sigma = T_i/T_{ef}, \qquad \beta = T_{ec}/T_{eh}, \qquad T_{ef} = T_{ec}T_{eh}/(\mu T_{ec} + \nu T_{eh}), \qquad \beta < 1.$$

In equations (1)–(4), the non-dimensional parameters n, u, p are the number density, velocity and pressure of the ions; ϕ is the electrostatic potential; n_{ec} and n_{eh} are number densities of cool and hot electrons respectively, with μ and ν their initial number densities. σ is the ion temperature. T_{ec} and T_{eh} are the temperatures of cool and hot electrons respectively. The velocity u has been normalized by $(K_B T_{ef}/m)^{1/2}$, m being the ion mass. The densities n, n_{ec} and n_{eh} are normalized by the equilibrium plasma density n_0. The space x and time t are normalized by $(4\pi e^2 n_0/K_B T_{ef})^{-1/2}$ and $(m/4\pi e^2 n_0)^{1/2}$ respectively, whereas the potential ϕ is normalized by $K_B T_{ef}/e$, K_B being the Boltzmann constant.

It is to be mentioned that if $\sigma = 0$ in equations (1)–(4), a set of basic equations is obtained which describe a system of collisionless plasmas composed of cold ions and two types of electrons (hot and respectively cold). These types of basic equations had been used by Buti

[25], Nejoh [26] and others for the study of ion-acoustic solitary waves in plasma. But, when we set $\mu = 1$, $\nu = 0$ and $\sigma \neq 0$ in equations (1)–(4), we obtain a system of basic equations governed by a collisional plasma with a mixture of warm ion-fluid and single component hot isothermal electron, which were used by Roychowdhury and Bhattacharya [27] for the study of ion-acoustic solitary waves in the absence of drift ions (i.e. $u_0 = 0$).

Now, for the present study, it is assumed that the basic equations are supplemented by the following boundary conditions as $|x| \longrightarrow \infty$,

$$n \longrightarrow 1, \qquad u \longrightarrow u_0, \qquad p \longrightarrow 1, \qquad \phi \longrightarrow 0. \tag{5}$$

To study the ion-acoustic solitary waves in the plasma by using Sagdeev's pseudopotential [28] approach, we make the variables depending on a single independent variable η defined by

$$\eta = x - Vt \tag{6}$$

where 'V' is the velocity of solitary wave.

Therefore, equations (1)–(3) yield

$$-V\frac{\mathrm{d}n}{\mathrm{d}\eta} + \frac{\mathrm{d}}{\mathrm{d}\eta}(nu) = 0 \tag{7}$$

$$(u - V)\frac{\mathrm{d}u}{\mathrm{d}\eta} + \frac{\sigma}{n}\frac{\mathrm{d}p}{\mathrm{d}\eta} = \frac{\mathrm{d}\phi}{\mathrm{d}\eta} \tag{8}$$

$$(u - V)\frac{\mathrm{d}p}{\mathrm{d}\eta} + 3p\frac{\mathrm{d}u}{\mathrm{d}\eta} = \frac{\mathrm{d}\phi}{\mathrm{d}\eta}. \tag{9}$$

Now, integrating (7) and using the boundary conditions (5) one can obtain

$$n = \frac{(V - u_0)}{(V - u)}. \tag{10}$$

Similarly, integrating (9) and using the same boundary conditions,

$$p = \frac{(V - u_0)^3}{(V - u)^3}. \tag{11}$$

Now, using (10) in (8) we obtain after integration

$$2\phi(u) = 2uV - u^2 - 2u_0V + u_0^2 + \frac{3\sigma pu}{2(V - u_0)} + 3\sigma\left(1 - \frac{pV}{V - u_0}\right). \tag{12}$$

Again, using (6) in (4) we get

$$\frac{\mathrm{d}^2\phi(u)}{\mathrm{d}\eta^2} = n_e - n. \tag{13}$$

Now, using equations (10), (11) and (12), equation (13) reduces to

$$\frac{\mathrm{d}^2\phi(u)}{\mathrm{d}\eta^2} = G(u), \tag{14}$$

where

$$G(u) = \mu\exp(\phi/\mu + \nu\beta) + \nu\exp(\beta\phi/\mu + \nu\beta) - \frac{(V - u_0)}{(V - u)} \tag{15}$$

and

$$\phi(u) = \frac{1}{2}\left[(V - u_0)^2 - (V - u)^2 + \frac{3\sigma u}{(V - u_0)}\left(\frac{V - u_0}{V - u}\right)^3 - 3\sigma\left\{1 - \frac{V}{(V - u_0)}\left(\frac{V - u_0}{V - u}\right)^3\right\}\right]. \tag{16}$$

Consequently,

$$\phi'(u) = (V-u) - \frac{3\sigma(V-u_0)^2}{(V-u)^3}. \tag{17}$$

Integrating equation (14) we find

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}u}\right)^2\left(\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)^2 = H(u) - K, \tag{18}$$

where

$$H(u) = 2\int G(u)\frac{\mathrm{d}\phi}{\mathrm{d}u} \tag{19}$$

and 'K' is an arbitrary integration constant.

Now, using (15) and (16), it is easy to integrate (19) and obtain

$$H(u) = 2\left[(\mu+\nu\beta)\left(\mu\cdot \mathrm{e}^{\frac{\phi}{\mu+\nu\beta}} + \frac{\nu}{\beta}\cdot \mathrm{e}^{\frac{\beta\phi}{\mu+\nu\beta}}\right) - (V-u_0)u - \sigma\left(\frac{V-u_0}{V-u}\right)^3\right]. \tag{20}$$

It is to be mentioned that the phase velocity of the solitary wave V is greater than the the velocity of the ion-acoustic wave, i.e. $V > u$.

3. Analytical study

For the ion-acoustic solitary wave in an isothermal two-temperature-electron plasma with a warm positive ion, the physically admissible solution of equation (18) is obtained with some special observations:

(1) $\left(\frac{\mathrm{d}\phi}{\mathrm{d}u}\right)^2\left(\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)^2$ must be non-negative,

(2) u and $\left(\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)$ must be bounded.

From the expressions of ϕ and G given in (12) and (13), it is easy to make the following observations:

Observation 1. $G(u_0) = 0$.

Observation 2. $\phi(u_0) = 0$.

Observation 3. If (i) $(V-u_0)^2 < 3\sigma, \phi'(u) < 0$ for $u_0 \leqslant u < V$, (ii) $(V-u_0)^2 > 3\sigma, \phi'(u) > 0$ for $u_0 \leqslant u < u'$ and (iii) $\phi'(u) < 0$ for $v' < u < V$, where u' is given by $\phi'(u') = 0$.

Observation 4. If (i) $(V-u_0)^2 < 3\sigma, \phi'(u) > 0$ for $V < u \leqslant u_0$, (ii) $(V-u_0)^2 > 3\sigma, \phi'(u) > 0$ for $V < u < u'$, and (iii) $\phi'(u) < 0$ for $u' < u \leqslant V$, where u' is given by $\phi'(u') = 0$.

Note. There exists no physical solution of equation (15) if $\phi'(u) < 0$ for $u_0 < V$ and $\phi'(u) > 0$ for $u_0 > V$.

Therefore, for the physical solution of equation (15) we need the following requirements.

Requirement 1. There exists $u_{\max}$ (or $u_{\min}$) such that

$$H(u_0) = H(u_{\max}) = K, \qquad \text{for} \quad u_0 < u_{\max} < u'$$
$$\text{or, } H(u_0) = H(u_{\min}) = K, \qquad \text{for} \quad u' < u_{\min} < u_0.$$

Requirement 2.

$H(u) > K$ for either $u_0 < u < u_{\max}$
or, $u_{\min} < u < u_0$.

Our next task is to obtain simple conditions (either necessary or sufficient) for the requirements to be fulfilled. We establish the following theorems.

Theorem 1. *Equation (18) will admit a real and bounded solution if and only if*
(i) $V > u_0 + \sqrt{1+3\sigma}$, *for* $V > u_0$ *and*
(ii) $u_0 > V + \sqrt{1+3\sigma}$, *for* $V < u_0$.

Proof. Part-I. Conditions (i) and (ii) are necessary. If requirement-2 is fulfilled, one has $H(u) > H(u_0)$ for $u = u_0 + \epsilon$, ϵ (>0), however small, an arbitrary number. It immediately follows that

$$H'(u_0) > 0. \tag{21}$$

Using observation-1 we get from (19) and (21),

$$G'(u_0) \cdot \phi'(u_0) > 0. \tag{22}$$

Since by virtue of observation-3 and observation-4,
$\phi'(u_0) > 0$ for $u \leqslant u < u'$ and $\phi'(u_0) < 0$ for $u' < u < u_0$, we obtain from (22), (15) and (17) after some calculation

$$V > u_0 + \sqrt{1+3\sigma}, \qquad \text{for} \quad V > u_0$$

and

$$u_0 > V + \sqrt{1+3\sigma}, \qquad \text{for} \quad V < u_0. \qquad \square$$

Proof. Part-II. Conditions (i) and (ii) together are sufficient. We assume that conditions (i) and (ii) of theorem 1 are to be satisfied. Then from conditions (i) and (ii), we obtain

$$G'(u_0) > 0, \qquad \text{for} \quad V > u_0 \tag{23a}$$

$$G'(u_0) < 0, \qquad \text{for} \quad V < u_0. \tag{23b}$$

From (23*a*) and (23*b*) using the observation-1, observation-3 and observation-4, we obtain

$$H''(u_0) > 0. \tag{24}$$

From inequality (24) and the fact that $H'(u_0) = 0$, it follows that

$$H(u_0 + \epsilon) > H(u_0) \tag{25}$$

for ϵ (>0), however small, an arbitrary number. □

Theorem 2. *A sufficient condition that equation (15) will admit a real and bounded solution is determined by* $H(u') - H(u_0) < 0$.

Proof. For a physically admissible solution it is obvious that $H'(u) \geqslant 0$ for $u = u_0 + \epsilon$, where ϵ (>0), however small, an arbitrary number. □

It follows that $H(u_0 + \epsilon) > H(u_0)$.

Also, from the conditions that

$$H(u') < H(u_0),$$

there exists a point $u_{\max}$ (say) (or $u_{\min}$ (say)) between u_0 and u' such that

$$H(u_{\max}) = H(u_0), \qquad \text{for} \quad u_0 < u < u_{\max}$$

or,

$$H(u_{\min}) = H(u_0), \qquad \text{for} \quad u_{\min} < u < u_0.$$

Consequently, there exists u^* between u_0 and $u_{\max}$ such that $H(u^*) = 0$.

3.1. Existence of solitary wave solution

Using equation (20) the condition $H(u') - H(u_0) < 0$ can be written as

$$\mu \cdot \frac{(e^p - 1)}{p} + (1-\mu) \cdot \frac{(e^{\beta p} - 1)}{\beta p} < \frac{2}{3}\left[1 + \frac{2}{(1+q)^2}\right] \qquad (26)$$

where

$$p = \frac{[(V-u_0)^2 - \sqrt{3\sigma}]^2}{2(\mu + \nu\beta)} \qquad \text{and} \qquad q = \left[\frac{3\sigma}{(V-u_0)^2}\right]^{\frac{1}{4}}.$$

Since $\beta < 1$ and $\mu < 1$, the above inequality is reduced to

$$\frac{(e^{\beta p} - 1)}{\beta p} < \frac{2}{3}\left[1 + \frac{2}{(1+q)^2}\right]. \qquad (27)$$

Substituting $u_0 = 0$ into (26), one gets

$$e^{\frac{(V-\sqrt{3\sigma})^2}{2}} - 1 - V^2\left[1 + \frac{\sigma}{V^2} - \frac{4}{3}\left(\frac{\sigma}{V^2}\right)^{\frac{1}{4}}\right] < 0. \qquad (28)$$

It is to be noted that (27) is the condition for the existence of a solitary-wave solution, obtained by Roychowdhury and Bhattacharyya [27].

Further, substituting $\sigma = 0$ into (27), we obtain

$$\left[e^{\frac{V^2}{2}} - 1 - V^2\right] < 0.$$

Now, $[e^{\frac{V^2}{2}} - 1 - V^2] < 0$ when $V < \kappa$, κ being the solution of $e^{\frac{V^2}{2}} = 1 - V^2$.

Taking $\kappa \approx 1.6$, we find $V < 1.6$.

Also, from inequality (i) of theorem 1,

$$V > 1 \qquad \text{for} \quad \sigma = 0, \quad u_0 = 0.$$

Therefore,

$$1 < V < 1.6.$$

The above condition was established by Sagdeev [28] for the existence of solitary wave solution in a non-drifting plasma.

Theorem 3. *For the given values of* σ, μ, ν *and* β*, a necessary condition for equation (15) to admit a physical and bounded solution is given by*

$$V - u_0 < \left[\sqrt{3\sigma} + \sqrt{\frac{2Z_0}{\beta}(\mu + \nu\beta)}\right],$$

Z_0 *satisfying the equation* $e^Z = 1 + 2Z$*; if the condition of the theorem 2, that is,* $H(u') - H(u_0) < 0$*, holds.*

Proof. Using (17) one can write the inequality $H(u') - H(u_0) < 0$ as

$$2\left[(\mu+\nu)\left\{\mu(e^p - 1) + \frac{\nu}{\beta}(e^{\beta p} - 1)\right\} - (V - u_0)\left\{1 + \frac{\sigma}{(V-u_0)^2} - \frac{4q}{3}\right\}\right] < 0 \qquad (29)$$

where $u' = (V - \sqrt{\sqrt{3\sigma}} \cdot \sqrt{V - u_0})$ is a solution of the equation $\phi'(u') = 0$. p and q have been defined earlier. Substituting

$$(V - u_0) - \sqrt{3\sigma} = X \qquad (30a)$$

and

$$\left(\frac{3\sigma}{(V-u_0)^2}\right)^{1/4} = Y, \tag{30b}$$

the inequality (28) reduces to

$$2\left[(\mu+\nu)\left\{\mu\left(e^{\frac{X^2}{2(\mu+\nu\beta)}}-1\right)+\frac{\nu}{\beta}\left(e^{\frac{\beta X^2}{2(\mu+\nu\beta)}}-1\right)\right\}-\frac{X^2}{3}\left\{1+\frac{2}{(Y+1)^2}\right\}\right]<0. \tag{31}$$

□

The necessary condition of the theorem will be established if we find the condition (or conditions) for which $[H(u') - H(u_0)]_{\min} < 0$ holds. Now, the minimum value of $[H(u') - H(u_0)]$ is

$$2\left[(\mu+\nu)\left\{\mu\left(e^{\frac{X^2}{2(\mu+\nu\beta)}}-1\right)+\frac{\nu}{\beta}\left(e^{\frac{\beta X^2}{2(\mu+\nu\beta)}}-1\right)1\right\}-X^2\right]$$

which is obtained by putting $Y = 0$, i.e. the least value of Y.

The inequality $[H(u') - H(u_0)]_{\min} < 0$ can be written as

$$\mu\cdot\frac{(e^Z-1)}{Z}+(1-\mu)\frac{(e^{\beta Z}-1)}{\beta Z}<2 \tag{32}$$

where

$$Z=\frac{X^2}{2(\mu+\nu\beta)}$$

$$X=(V-u_0)^2-\sqrt{3\sigma}.$$

Let us choose $Z < Z_0/\beta$, where Z_0 is a solution of $e^Z = 1 + 2Z$.

Then, it follows that the inequality (31) holds provided

$$Z<\frac{Z_0}{\beta}. \tag{33}$$

Using (29) and (32), one can obtain from (29)

$$V-u_0<\left[\sqrt{3\sigma}+\sqrt{\frac{2Z_0}{\beta}(\mu+\nu\beta)}\right].$$

Hence, the condition is necessary.

Note. $V - u_0 < [\sqrt{3\sigma}+\sqrt{\frac{2Z_0}{\beta}(\mu+\nu\beta)}]$ is a necessary condition of $H(u') - H(u_0) < 0$, but not sufficient.

Theorem 4. *A sufficient condition for equation (18) to admit a physical and bounded solution is that for fixed values of β (<1) and sufficiently small values of μ (<1)*

$$\mu\cdot\frac{(e^Z-1)}{Z}+(1-\mu)\frac{(e^{\beta Z}-1)}{\beta Z}<\frac{2}{3}\left[1+\frac{2}{(1+Y)^2}\right],$$

if the condition of theorem-2, that is, $H(u') - H(u_0) < 0$ holds.

Proof. The condition $H(u') - H(u_0) < 0$ can be written as

$$\mu\cdot\frac{(e^Z-1)}{Z}+(1-\mu)\frac{(e^{\beta Z}-1)}{\beta z}<\frac{2}{3}\left[1+\frac{2}{(1+Y)^2}\right]. \tag{34}$$

Since $\beta < 1$ and $\mu < 1$, then $\frac{(e^{\beta Z}-1)}{\beta z} < \frac{(e^{Z}-1)}{Z}$. Therefore, from (34) it follows that

$$\frac{(e^{\beta Z}-1)}{\beta Z} < \frac{2}{3}\left[1+\frac{2}{(1+Y)^2}\right]. \tag{35}$$

□

We see that if $Y < 1$, the inequality (35) holds for $\beta \to 0$. Consequently, for sufficient small values of β, the inequality (35) holds good. This means that the inequality (34) holds for $\mu = 0$, if one fixes the sufficiently small value of β.

Therefore, for a sufficiently small value of μ, the inequality (34) holds for fixing the values of β.

Note that

$$\mu \cdot \frac{e^{Z}-1}{Z} + \frac{\nu}{\beta Z} < \frac{2}{3}\left[1+\frac{2}{(1+Y)^2}\right]$$

is a sufficient condition of $[H(u') - H(u_0)] < 0$.

4. Discussion and conclusion

In the present investigation, we have theoretically investigated the ion-acoustic solitary waves in a one-dimensional model of plasma consisting of an ion component, drifting with some velocity, and a two-temperature electron component. The requirements for the existence of a solitary solution are derived for this model. The conditions for the existence of real and bounded solutions of equation (18) are obtained in an exact form of the electrostatic potential (ϕ). The necessary conditions are given by theorems 1 and 3 while theorems 2 and 4 give simpler sufficient conditions. We see from theorem 1 that the phase velocity (V) satisfies two inequalities for $V > u_0$ and $V < u_0$. From the inequality $H(u') - H(u_0) < 0$ we have established the existence of the physically admissible solitary wave solution for a plasma consisting of two-temperature electrons and drift ions. To see the effects of u on H for different values of μ, β, ϕ, σ and u_0, graphs are plotted as shown in figures 1–5 considering a model plasma. In figure 1, it is seen that $H(u)$ decreases as u increases and $H(u)$ is large for large values of μ. Figure 2 shows that $H(u)$ also decreases with the increase of u, but $H(u)$ for $\beta = 0.36$ is smaller than its value for $\beta = 0.18$. Again, when β is larger than 0.36, $H(u)$ increases and finally for $\beta = 0.9H(u)$ attains a value which is larger than its value for $\beta = 0.18$. In figure 3 the variation of $H(u)$ with u for different values of σ is shown. The figure shows a continuous decrease of $H(u)$ with the increase of u for a small value of σ. For large values of β, however $H(u)$ decreases with the increase of u up to a certain value and then starts to increase with the increase of u. The effects of ϕ on the variation of $H(u)$ with u are shown in figure 4. It is observed that $H(u)$ is large for large values of ϕ. Moreover, $H(u)$ decreases with the increase of u. The variations of $H(u)$ with u for different values of u_0 are shown in figure 5. It is seen that $H(u)$ decreases with the increase of u and $H(u)$ is large for large values of u_0.

To get some ideas about the lower and upper limits of the phase velocity V, numerical estimations are made considering a model plasma. The numerical results are given in tables 1, 2 and 3. It is observed that the upper and lower values of the phase velocity V of the wave in a drift plasma are much different from the values obtained by earlier authors in case of nondrifting plasma. Neglecting the drift motion of the ions (i.e. $u_0 = 0$) and considering the presence of only one component electron in the plasma, the values of the upper and lower limits of the phase velocity are the same as obtained by Roychowdhury and Bhattacharya [27]. Further, for $\sigma = 0$, the limiting value of the phase velocity V reduces

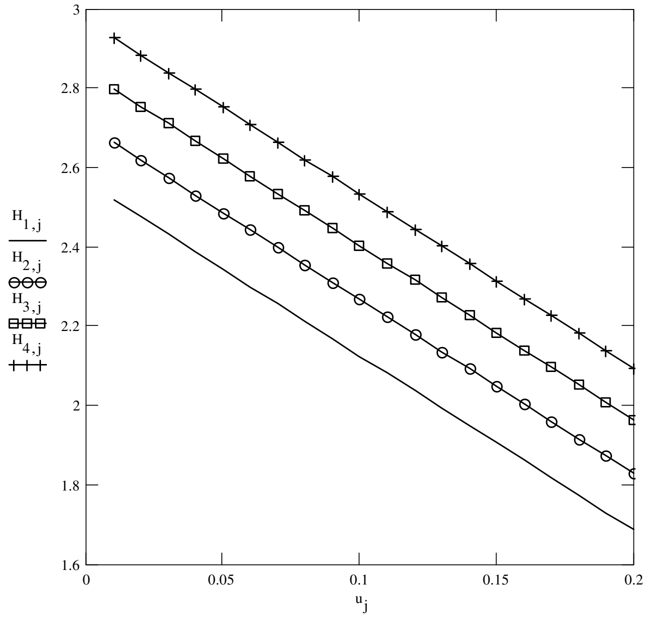

Figure 1. Variation of H for different values of u and μ: solid line for $\mu = 0.02$, o's line $\mu = 0.04$, box's line for $\mu = 0.06$ and +'s line for $\mu = 0.08$. The values of other parameters of the plasma are: $\sigma = 0.01$, $\beta = 0.2$, $\phi = 0.001$, $V = 2.6$, $u_0 = 0.4$.

Table 1. The limiting values of V for different μ.

μ	Limiting values of V
0.03	$1.437 < V < 2.661$
0.06	$1.437 < V < 2.993$
0.09	$1.437 < V < 3.284$
0.12	$1.437 < V < 3.545$
0.15	$1.437 < V < 3.785$
0.18	$1.437 < V < 4.007$
0.21	$1.437 < V < 4.216$
0.24	$1.437 < V < 4.413$
0.27	$1.437 < V < 4.6$
0.3	$1.437 < V < 4.778$

$\sigma = 1/40$, $\beta = 1/20$, $u_0 = 0.4$, $z_0 = 1.257$.

Table 2. The limiting values of V for different σ.

σ	Limiting values of V
1/10	$1.44 < V < 4.515$
1/20	$1.372 < V < 4.355$
1/30	$1.349 < V < 4.284$
1/40	$1.337 < V < 4.241$
1/50	$1.33 < V < 4.212$
1/60	$1.325 < V < 4.191$
1/70	$1.321 < V < 4.174$
1/80	$1.319 < V < 4.161$
1/90	$1.317 < V < 4.15$
1/100	$1.315 < V < 4.141$

$\mu = 0.15$, $\beta = 1/30$, $u_0 = 0.3$, $z_0 = 1.257$.

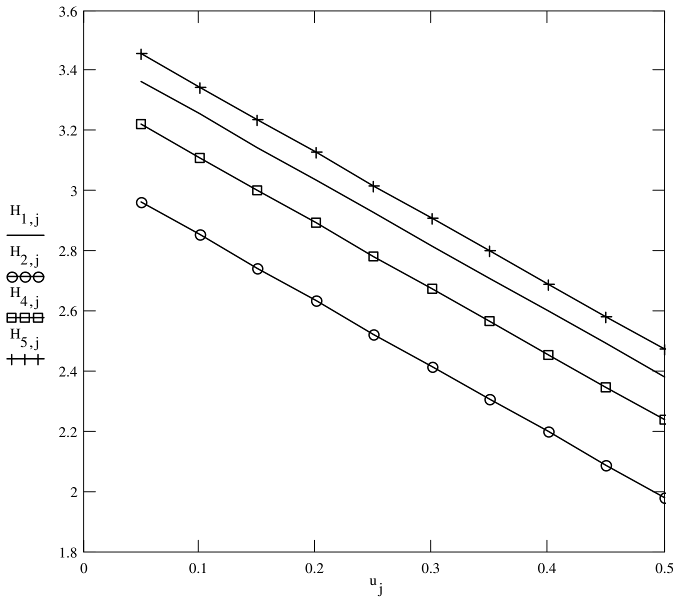

Figure 2. Variation of H for different values of u and β: solid line for $\beta = 0.18$, o's line $\beta = 0.36$, box's line for $\beta = 0.72$ and +'s line for $\beta = 0.9$. The values of other parameters of the plasma are: $\sigma = 0.01$, $\mu = 0.15$, $\phi = 0.001$, $V = 1.8$, $u_0 = 0.7$.

Table 3. The limiting values of V for different β.

β	Limiting values of V
1/10	$1.449 < V < 3.147$
1/20	$1.449 < V < 3.827$
1/30	$1.449 < V < 4.384$
1/40	$1.449 < V < 4.866$
1/50	$1.449 < V < 5.298$
1/60	$1.449 < V < 5.692$
1/70	$1.449 < V < 6.058$
1/80	$1.449 < V < 6.4$
1/90	$1.449 < V < 6.723$
1/100	$1.449 < V < 7.029$

$\mu = 0.15$, $\sigma = 1/30$, $u_0 = 0.4$, $z_0 = 1.257$.

to the Sagdeev's formula [27]. It is to be noted that the necessary conditions are obtained in terms of the velocity of ion-acoustic wave u instead of the number density of the ions (n) or the electrostatic potential ϕ, because the drift velocity of the ions (u_0) is considered in the plasma. However, the necessary and sufficient conditions in terms of the density of ions or electrostatic potential may be obtained following our present analysis. One may find that the works of Kourakis and Shukla [18] are different from our analysis. Modulation of ion-acoustic waves and excitation of envelope solitons in a two-electron-temperature plasma are studied by Kourakis and Shukla [18]. But, the motivation of the present study is to obtain the necessary and sufficient conditions for the excitation of ion-acoustic solitary waves in a plasma consisting of two-temperature electrons and warm drift ions. In this regard, it is to be mentioned that

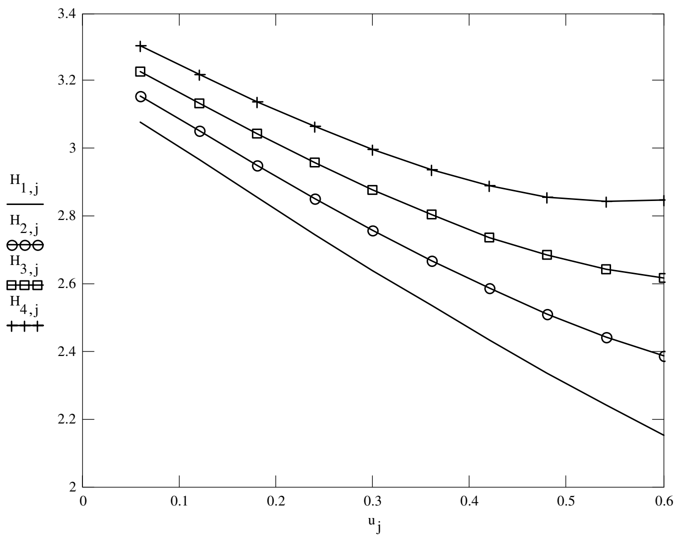

Figure 3. Variation of H for different values of u and σ: solid line for $\sigma = 0.02$, o's line $\sigma = 0.04$, box's line for $\sigma = 0.06$ and +'s line for $\sigma = 0.08$. The values of other parameters of the plasma are: $\mu = 0.1$, $\beta = 0.5$, $\phi = 0.001$, $V = 2.6$, $u_0 = 0.4$.

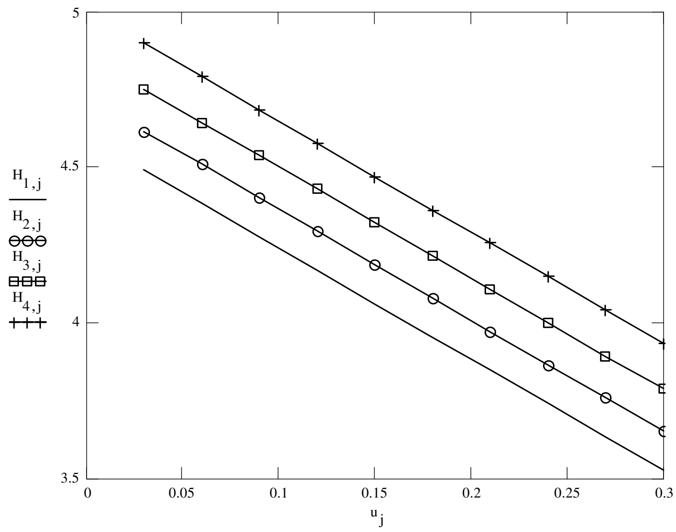

Figure 4. Variation of H for different values of u and ϕ: solid line for $\phi = 0.18$, o's line for $\phi = 0.36$, box's line for $\phi = 0.72$ and +'s line for $\phi = 0.9$. The values of other parameters of the plasma are: $\mu = 0.15$, $\beta = 0.1$, $\sigma = 0.03$, $V = 2.6$, $u_0 = 0.8$.

drift velocity of the ions has important role when ion-acoustic solitary waves are studied in relativistic plasma. Das and Paul [29] showed that the relativistic effect would be introduced on the ion acoustic solitons only in the presence of streaming of ions in the plasma. The effect of magnetic field on the excitation of ion-acoustic solitary waves in plasma consisting of drift ions and two-temperature electrons would give interesting results particularly under

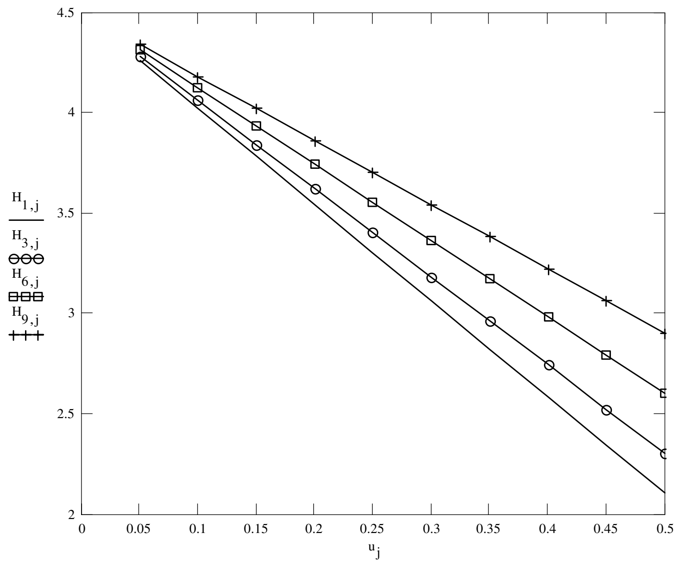

Figure 5. Variation of H for different values of u and u_0: solid line for $u_0 = 0.1$, o's line $u_0 = 0.3$, box's line for $u_0 = 0.6$ and +'s line for $u_0 = 0.9$. The values of other parameters of the plasma are: $\sigma = 0.001$, $\mu = 0.15$, $\beta = 0.1$, $\phi = 0.01$, $V = 2.5$.

Table 4. The limiting values of V for different u_0.

u_0	Limiting values of V
0.1	$1.137 < V < 4.041$
0.2	$1.237 < V < 4.141$
0.3	$1.337 < V < 4.241$
0.4	$1.437 < V < 4.341$
0.5	$1.537 < V < 4.441$
0.6	$1.637 < V < 4.541$
0.7	$1.737 < V < 4.641$
0.8	$1.837 < V < 4.741$
0.9	$1.937 < V < 4.841$
1.0	$2.037 < V < 4.941$

$\mu = 0.15$, $\sigma = 1/40$, $\beta = 1/30$, $z_0 = 1.257$.

the necessary and sufficient conditions. Theoretical investigation on the ion-acoustic solitary wave considering these parameters is in progress and would be communicated shortly.

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